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Evolutionary topology optimization for temperature reduction of heat conducting fields

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Abstract

This paper aims at developing an efficient finite element based computational procedure for the topology design of heat conducting fields. To evaluate the temperature change in a specific position, due to varying the conducting material distribution in other regions, a discrete temperature sensitivity is derived for an evolutionary topology optimization method. In the topology optimization of the conducting fields, the thermal conductivity of an individual finite element is considered as the design variable. By removing or degenerating the conductive material of the elements with the most negative sensitivity, the temperature objective at the control point can be most efficiently reduced. Illustrative examples are presented to demonstrate this proposed approach.

Keywords: Topology optimization; Sensitivity analysis; Finite element; Heat transfer; Conductive field

1. Introduction

It has been recognized that computational design optimization can yield substantial improvements in the design of thermal systems such as a wing box [1], a thermal diffuser [2] and cooling fins [3,4]. One of the most frequently encountered design problems in thermal engineering is to find a best possible geometrical shape or size that can achieve specific heat objectives [5]. Conventionally, the computational design is carried out by an iterative procedure consisting of; finite (or boundary) element heat solver; sensitivity analysis and optimization with mathematical programming. It is worth noting that the existing literature has a strong focus on the evaluation of various thermal-related sensitivities with respect to size or shape design variables. This has been pioneered by a number of researchers since the 1980s [6]. Haftka adopted a finite element based discrete technique for computing the sensitivities of steady-state and transient fields with respect to the changes in design parameters [7]. Meric, constructed the shape design sensitivities for numerical optimization of heat conducting solids [8,9]. Dems [10] and Tortorelli et al. [11] derived the sensitivities for both linear and non-linear thermal systems by using the Lagrangian multiplier and adjoint variable techniques. Park and Yoo developed a sensitivity-based algorithm for size and shape problems within

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Nomenclature

$\boldsymbol{C}$	global conductivity matrix
$\Delta\boldsymbol{C}$	the change in global conductivity matrix due to perturbation $\Delta\eta$
$\boldsymbol{C}^{\text{new}}$	global conductance matrix in new system
$\Delta\boldsymbol{C}^{e}$	the change in the elemental conductance matrix of the eth element
$\boldsymbol{C}^{e}$	the elemental conductance matrix at element e
D	element set in design domain
E	element set in entire heat field
I	the volume distribution function of insulator material
J_{O}	the objective function in terms of nodal temperature
N	element set in non-design domain
Q	heat energy generated per unit volume
$\boldsymbol{Q}$	applied heat load vector
$\boldsymbol{Q}_j$	fictitious load in thermal system
$\boldsymbol{T}$	global nodal temperature vector
$\boldsymbol{T}_j$	resulting temperature vector of fictitious system
T_j	the temperature at the jth node
T_H	the temperature at the heat sources
$\boldsymbol{T}^{e}$	the temperature vector of the eth element under real load $\boldsymbol{Q}$
$\boldsymbol{T}_j^{e}$	the temperature vector of the eth element under fictitious load $\boldsymbol{Q}_j$
$\Delta\boldsymbol{T}$	the change in global temperature vector
ΔT_j	the change in the jth temperature component
T_{Γ}	the temperature at boundary of the field
V_0	the volume constraint of insulation material
V_1, V_2	the volumes of insulation and conduction materials respectively

Greek symbols

$\hat{\alpha}_{\mathrm{T}}^{e} \geqslant 0$	the translated sensitivity of element e
α_0	a positive constant to translate all sensitivity to positivity
α_{T}^{e}	the temperature sensitivity of the eth element
η	the design variable of each candidate element
κ_1	the insulator with a void or a very low conductivity
κ_2	conductivity of conductive material
κ_x, κ_y, κ_z	heat conductivities in x, y and z directions (W/mm K)
μ^{k}	*Alteration ratio* of the evolutionary procedure
$\Delta\mu$	*Evolutionary Rate* of the ESO procedure
τ	the convergence tolerance of the objective function
$\Delta\upsilon_{\mathrm{f}}$	fixed removal ratio of the material volume
$\Delta\upsilon_{\mathrm{v}}$	varying removal ratio of the material volume
Ω	heat analysis domain

a boundary element framework [2]. Saigal and Chandra utilized the implicit direct differentiation of discrete boundary integral equations for the sensitivity and optimization of the heat diffusion problems [12]. Lee also proposed a direct differentiation method for axi-symmetric thermal conducting solids by boundary element method [13]. To facilitate general non-linear programming algorithms, Hou and Sheen presented a numerical technique to computing the second-order sensitivities for heat conduction problems [3]. More recently, by reducing the non-linear thermal equation to standard Laplace problem, Sluzaec and Kleiber derived the non-linear steady-state design sensitivities of diffuser's material in terms of the adjoint method [14]. Meric also developed the sensitivities for general non-linear conductive problems [15]. Dems and Rousselet derived the continuum shape sensitivities for anisotropic materials [16]. By using curvilinear grid generation and conjugate gradient methods, Lan et al presented some design problems of the shape profiles of the conductive medium towards a uniform temperature distribution [5]. Gu et al presented a discrete form of sensitivity analysis for linear and non-linear transient heat conduction problems by means of a so-called precise time integration method [17].

In all the above-mentioned work, a common point has been to compute the shape-based sensitivities and then optimize the geometric boundaries by changing shape parameters (e.g. nodal coordinates of interfaces or boundaries). Although these shape or size based designs are of great theoretical significance, there exist several limitations in practical applications. Firstly, a complex geometry needs being represented in an efficient manner to accommodate an accurate sensitivity analysis [5]. In other words, the sensitivity analysis usually relies on the scheme of boundary or interface representation. Secondly, a sophisticated re-meshing process is often required when the changing mesh suffers serious distortion after a number of iterations [18]. Finally, the designs are somewhat restricted by the initial guess of the shape and there is less possibility to seek an innovative optimal topology.

In the past two decades, substantial efforts have been devoted to the development of some efficient and robust topology optimization procedures for a full range of problems in continuum and structural mechanics [19–

24]. It has been noticed that the optimum topology designs can provide a more sizeable improvement in structural performance than optimal size and shape designs. Such topology optimization is especially beneficial at the conceptual design phase of a product [24]. To migrate the well-established topology algorithms from elastic mechanics problems to thermal problems, Li et al presented a unified non-gradient procedure to both shape and topology optimizations for heat conduction problems, in which a uniform efficiency of material usage in terms of local heat flux was achieved [25]. Such a flux-based topology optimization procedure was later extended to other physical situations [26], where the design optimizations covered a range of practical examples in torsional, conductive, electrical and magnetic fields that are governed by the quasi-harmonic equation.

To enable more specific design objectives, one of the crucial issues associated with topology optimization is to evaluate the topological sensitivities. Such a problem has attracted a certain attention recently. Motivating from the typical inverse problem of identifying a small inclusion, Sokolowski and Zochowski presented topological derivatives for a general 3D Laplace equation [27]. They formulated the flux-type objective functions and theoretically claimed some potential in the designs of diffusion or heat transfer problems. Guillaume and Idris derived topological sensitivities of Dirichilet problem by using an asymptotic expansion technique [28]. Recently, the authors also derived a discrete temperature sensitivity with respect to the presence and absence of a candidate element for the temperature control problem and some innovative topologies were presented in the heat conductive fields [29]. Turteltaub employed solid isotropic material with penalization (SIMP) method for the bi-material redistributions in a transient heat conduction problem [30]. The temperature field at a given time step is sought as close as possible to a target distribution in a least-square sense. Based on the well-established shape sensitivity concepts, Novotny et al. developed a topological derivative for ordinary partial differential equations, in which the heat transfer with both Dirichlet (temperature) and Neuman (heat flux) boundaries are considered in the newly created holes. In their work, some energy-type cost functions were set for the design objectives and material removal is carried out in the design process [31]. Starting from some seed holes, Barbarosie presented a reshape process that produced some interesting periodic inclusions of non-conductive materials or voids to yield a certain heat conductive properties [18].

Relatively speaking, however, the methodology and applications of the topological designs in thermal conducting media have been less addressed comparing with the structural mechanics problems in spite its potential significances. On the basis of previous relevant work by the authors [32–34], this paper aims at extending the evolutionary structural optimization (ESO) method [23,35,36] to the material topology designs of the thermal conduction problems. In this study, the ESO method is employed to minimize the objective temperature at a specific node by both topology (conducting solid and void) and bi-material (conductor and insulator) design models. To estimate the effect of conductive material's removal or degeneration on the control temperature, a discrete temperature sensitivity is derived for steady state heat conducting fields. To demonstrate the capabilities of the method presented herein, several numerical examples are also presented in this paper.

2. Sensitivity analysis for steady-state heat fields

The governing equation for general steady state heat conduction problems is well known as,

$$\frac{\partial}{\partial x}\left(\kappa_x\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\kappa_y\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\kappa_z\frac{\partial T}{\partial z}\right)+Q=0, \tag{1}$$

where κ_x, κ_y and κ_z denote heat conductivities, Q the heat energy generated per unit volume. Eq. (1) can be approximated by means of a finite element formulation as [37]

$$\boldsymbol{CT}-\boldsymbol{Q}=\boldsymbol{0}, \tag{2}$$

where $\boldsymbol{C}$ represents the global conductivity matrix, $\boldsymbol{T}$ the global nodal temperature vector and $\boldsymbol{Q}$ the applied heat load vector.

In a topological optimization, the materials of conductive field are systematically redistributed among those candidate elements in design domain (represented by $\mathbf{D}$) so that a certain form of objective function can be sought. The present study concentrates on temperature reduction by optimizing the material configuration. From the perspectives of engineering applications, it is usually expected that the final designs can be one of the following two cases with: (1) some voids or holes created in the conductive fields and (2) some insulators embedded into the conductive materials. The appropriate holes or insulators will play a role on redistributing the conductive fields thereby achieving the certain value for the prescribed design objective. It is desirable that the materials used in the final designs should be either the conductor with a high conductivity of κ_2 or the insulator with either a void ($\kappa_1 = 0$) or a very low conductivity of κ_1 ($\kappa_1 \ll \kappa_2$).

Mathematically, the topology optimization problems for the temperature control can be stated as follows,

$$\begin{cases} J_{\mathrm{O}}=T_j(\eta), \\ \text{s.t. } V_0-\int_\Omega \mathbf{I}(\eta(x))\,\mathrm{d}\Omega \geqslant 0, \end{cases} \tag{3}$$

where $J_{\mathrm{O}} = T_j(\eta)$ denotes the objective function in terms of temperature at controlled node j, Ω the domain of heat analysis, $\mathbf{I}$ the volume distribution function of insulator material and V_0 the volume constraint for insulator materials. $\eta \in [0,1]$ represents the design variable of each candidate element $e \in \mathbf{D}$ and the corresponding conductivity of material can be expressed in a linear fashion as,

$$\kappa(\eta) = (1-\eta)\kappa_1 + \eta\kappa_2. \tag{4}$$

For the full conductive materials, $\eta = 1$ and for the void or full insulation materials, $\eta = 0$.

It is assumed that the initial design domain is fully occupied by conductive materials. It is expected that, by removing or degenerating some conductive elements, newly created cavities or material distribution can have the objective temperature reduced. To do so, the effect of the element removal or material deterioration on the objective temperature J_O needs to be assessed. This can be done by either additional exact analyses or employing some approximate procedure to reduce the number of costly exact analyses [20]. By comparing the relative effects, the most suitable elements to be altered can be identified [38] and the removal or degeneration of such conductive elements will result in a most efficient reduction in the objective temperature.

The alteration (removal or degeneration) of a conductive element (by a perturbation $-\Delta\eta$ in the design variable) will lead the global conductance matrix of system (2) to changing by

$$\Delta \boldsymbol{C} = \boldsymbol{C}(\boldsymbol{\eta}) - \boldsymbol{C}^{\text{new}}(\boldsymbol{\eta} - \Delta\boldsymbol{\eta}), \tag{5}$$

where $\boldsymbol{C}^{\text{new}}$ stands for the new conductance matrix of the resulting field after altering the eth element and $\boldsymbol{\eta}$ the vector of design variables in the old system. For simplicity, assume that the variation of the element has no effect on the heat load vector $\boldsymbol{Q}$, i.e. $\Delta\boldsymbol{Q} = \mathbf{0}$. The equilibrium condition for the new system of conducting field is given by

$$(\boldsymbol{C} - \Delta\boldsymbol{C})(\boldsymbol{T} + \Delta\boldsymbol{T}) = \boldsymbol{Q}. \tag{6}$$

Subtracting Eq. (2) from (6) and ignoring the higher order term, one can compute the change of the temperature vector as,

$$\Delta\boldsymbol{T} = \boldsymbol{C}^{-1}\Delta\boldsymbol{C}\boldsymbol{T}. \tag{7}$$

To identify the change in a specific jth temperature component T_j, a fictitious load $\boldsymbol{Q}_j$, in which the jth component is equal to unity and all the others are equal to zero, is introduced. Multiplying Eq. (7) by $\boldsymbol{Q}_j$, the change ΔT_j in the specific temperature component due to altering the eth element, as shown in Fig. 1, is determined by

$$\Delta T_j = \boldsymbol{Q}_j^{\text{T}}\Delta\boldsymbol{T} = \boldsymbol{Q}_j^{\text{T}}\boldsymbol{C}^{-1}\Delta\boldsymbol{C}\boldsymbol{T} = \boldsymbol{T}_j^{\text{T}}\Delta\boldsymbol{C}\boldsymbol{T}, \tag{8}$$

where $\boldsymbol{T}_j^{\text{T}}$ denotes the solution of the fictitious system,

$$\boldsymbol{C}\boldsymbol{T}_j - \boldsymbol{Q}_j = \mathbf{0}. \tag{9}$$

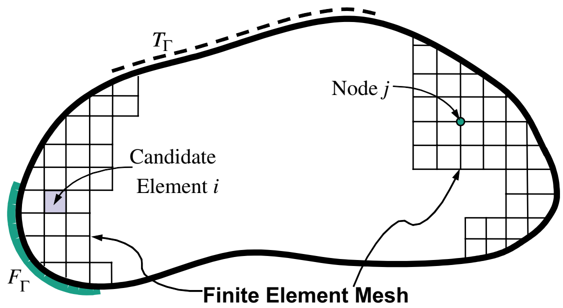

Fig. 1. Finite element model for sensitivity analysis.

In fact, the temperature change ΔT_j can be readily calculated at one element level as

$$\Delta T_j = \boldsymbol{T}_j^{e\text{T}}\Delta\boldsymbol{C}^e\boldsymbol{T}^e, \tag{10}$$

where $\Delta\boldsymbol{C}^e$ denotes the change in the elemental conductance matrix of the eth element due to perturbation $\Delta\eta$ in its the design variable, $\boldsymbol{T}_j^e$ and $\boldsymbol{T}^e$ denote the temperature vectors of the eth element under the fictitious load $\boldsymbol{Q}_j$ and the real load $\boldsymbol{Q}$ respectively. Represent the element set in whole conductive field by $\mathbf{E}(\mathbf{D} \subseteq \mathbf{E})$, for $e \in \mathbf{E}$, the value

$$\alpha_{\text{T}}^e = \boldsymbol{T}_j^{e\text{T}}\Delta\boldsymbol{C}^e\boldsymbol{T}^e \tag{11}$$

is defined as the temperature sensitivity of the eth element, which is used to estimate the temperature change at the jth degree of freedom due to the alteration of the conductivity of element e.

It should be noted that α_{T}^e can be either positive or negative, which implies that T_j may be changed in an opposite direction. In other words, the sensitivity numbers divide the design domain $\mathbf{D}$ into two types of regions, respectively positive $\mathbf{D}^+$ or negative $\mathbf{D}^-$. It is obvious that, to remove or degenerate the conductive materials from the negative sensitivity elements $e \in \mathbf{D}^-$ will result in the controlled temperature reduced steadily.

3. Evolutionary procedure

In this paper, two evolutionary procedures are presented. One is to set the initial design variables at the maximum possible value $\eta = 1$ or $\kappa(\eta = 1) = \kappa_2$, then gradually fully remove the material from those conductive elements with the most negative sensitivities. In other words, the design variables are altered to $\eta = 0$ or $\kappa(\eta = 0) = \kappa_1 = 0$ as in the typical ESO procedure [23,25]. The other is also set the initial design variable at $\eta = 1$ (i.e. $\kappa(\eta = 1) = \kappa_2$) for each candidate element, then progressively downgrade the conductive material to the insulation value, $\kappa(\eta) = \kappa_1 \ll \kappa_2$, for those most negatively sensitive elements. As a result of these two procedures, some holes or insulators are generated in the conductive domain, which will play a most efficient role on thermal shielding if the effects of convection and radiation are not considered. These two different approaches are described separately in the following sections.

3.1. Element removal

In the typical ESO topological design, elements are removed from the finite element model. Subsequently, the conductance matrix of whole system changes, in an element level, by

$$\Delta \boldsymbol{C}^e = \boldsymbol{C}^e, \tag{12}$$

where $\boldsymbol{C}^e$ denotes the elemental conductance matrix. For such a situation, the temperature sensitivity in Eq. (11) can be therefore expressed as

$$\alpha_{\mathrm{T}}^e = \boldsymbol{T}_j^{e\mathrm{T}} \boldsymbol{C}^e \boldsymbol{T}^e, \tag{13}$$

which indicates the temperature variation at node j due to the removal of element e from the conductive field.

3.2. Degeneration of conductive materials

In an evolutionary material design, the conductive material can be degenerated in such a way from a higher conductivity $\kappa(\eta)$ to a lower conductivity $\kappa(\eta - \Delta\eta)$ that the target temperature is gradually reduced. This has been accomplished by using a discrete descending stepwise design variables $\eta \in [\eta_1, \eta_2, \ldots, \eta_r]$ (here $\eta_1 = 1 > \eta_2 > \cdots > \eta_r = 0$) in previous development of the morphing ESO [23]. However, the method may lead to some semi-conductor materials remained in the final design domain though they do not usually dominate the design. Obviously, such a result could become less realistic in practical engineering applications. For this reason, design variable $\eta = 1$ or 0 is set in this study, in which $\kappa(\eta = 1) = \kappa_2$ is implicitly given for the high conductive material (conductor) and $\kappa(\eta = 0) = \kappa_1$ for the low conductive material (insulator).

As a result of degeneration of the conductive materials, the change in elemental conductance matrix is computed as,

$$\Delta \boldsymbol{C}^e = \boldsymbol{C}^e(\eta) - \boldsymbol{C}^e(\eta - \Delta\eta). \tag{14}$$

3.3. Evolutionary procedure

After subdividing the conductive field with a dense finite element mesh, the evolutionary procedure starts from a status with fully populated conductive elements ($\kappa(\eta = 1) = \kappa_2$). In the typical ESO procedure, the relative magnitudes of element's sensitivities determine the sequence of element's deletion or degeneration [23,35]. It would make the evolutionary procedure more controllable if all the temperature sensitivities of elements are translated to positive values. This can be accomplished by simply adding a certain positive constant number α_0, which is large enough so as to shift all sensitivity numbers greater than zero, i.e.

$$\hat{\alpha}_{\mathrm{T}}^e = \alpha_{\mathrm{T}}^e + \alpha_0 \geqslant 0. \tag{15}$$

It is clear that removing or degenerating those conductive elements which have the most negative value of α_{T}^e will result in the most significant contribution to the reduction of the target temperature. The evolutionary criterion for such a purpose is determined by comparing the temperature sensitivity of each element with the highest value $\hat{\alpha}_{\mathrm{T}}^{\max}$. That is to say that the element e is removed or degenerated from **D** (by assigning $\eta = 0$), if its sensitivity satisfies both

$$\alpha_{\mathrm{T}}^e < 0, \tag{16}$$

and

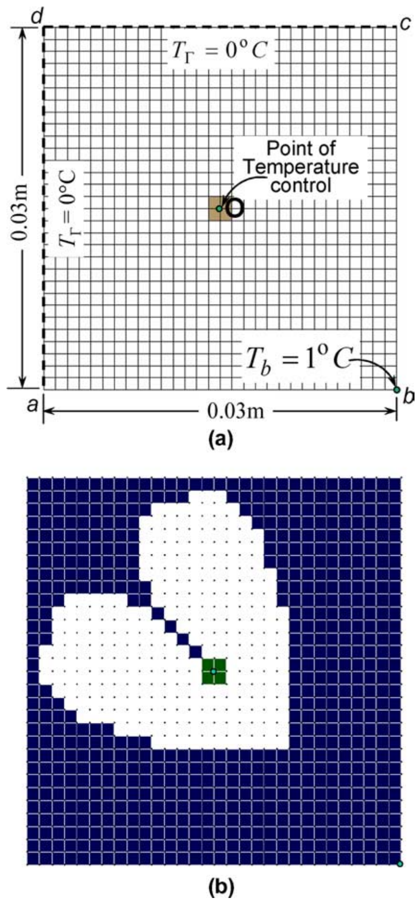

Fig. 2. Optimal design of a heat conductive field: (a) finite element modeling and (b) optimum topology.

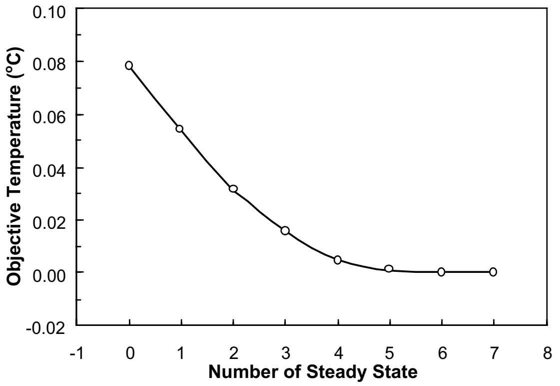

Fig. 3. Evolutionary history of the objective temperature at the controlled point.

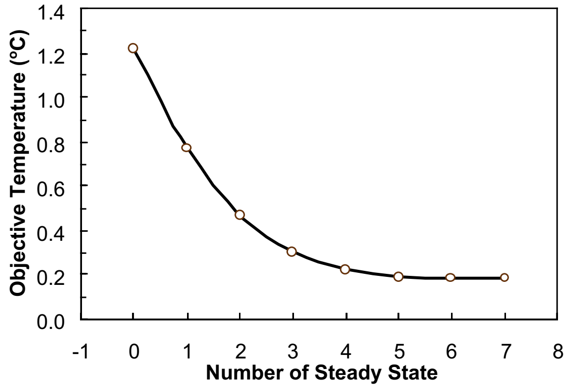

Fig. 5. The evolutionary history of objective temperature with two heat sources.

$$\hat{\alpha}_{\mathrm{T}}^{e} \leqslant \mu^{(k)} \times \hat{\alpha}_{\mathrm{T}}^{\max}, \tag{17}$$

where $\mu^{(k)}$ is termed as *Alteration ratio*, which is adopted to determine a threshold for the negativity of the temperature sensitivity. The process of the element alteration is repeated using the same value of $\mu^{(k)}$ until a *Steady State* is reached, which means that there are no more elements being deleted or degenerated at the current iteration. At this stage an *Evolutionary Rate* ($\Delta\mu$) is introduced so that

$$\mu^{(k+1)} = \mu^{(k)} + \Delta\mu^{(k+1)}. \tag{18}$$

For simplicity, the evolutionary rate $\Delta\mu$ is usually set as a constant throughout the evolving process, though a variable rate can be used [36]. In the following design examples, both the initial values of the alteration ratio $\mu^{(k=0)}$ and $\Delta\mu$ are set to 2% to ensure a smooth change between two steady states [23]. With the increased alteration ratio ($\mu^{(k+1)}$) the cycle of finite element analysis and element alteration takes place again until a new steady state is reached.

In order to end up the evolving procedure, two termination criteria are employed in this study. Firstly, when the evolving process has progressed to a certain degree, it is possible that all elemental sensitivities over the conductive design domain $\forall e \in \mathbf{D}^{(k)}$ becomes non-negativity, i.e.

$$\alpha_{\mathrm{T}}^{e} \geqslant 0 \tag{19}$$

or $\mathbf{D}^{-(k)} = \emptyset$, which implies that there is no further room to remove or degenerate the conductive materials. Hence the evolving process should be terminated at this stage. The non-negativity condition is also viewed as an optimality criterion by several other researchers [39,40].

Secondly, as more and more elements with highly negative sensitivities are removed or degenerated, the

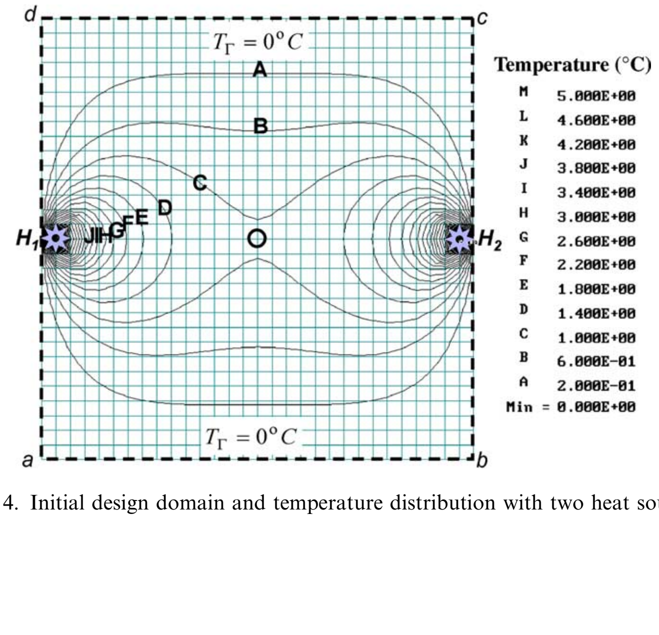

Fig. 4. Initial design domain and temperature distribution with two heat sources.

effect of element alteration on the target temperature becomes less and less significant. In this study, a tolerance τ (e.g. 10^{-6}) [23,32] is prescribed for the convergence check on the objective function (J_{O}). If the relative change in the objective temperatures in two successive iterations is less than the given tolerance τ, i.e.

$$\left|\frac{J_{\mathrm{O}}^{(k+1)} - J_{\mathrm{O}}^{(k)}}{J_{\mathrm{O}}^{(k)}}\right| \leqslant \tau \tag{20}$$

then it is deemed that a convergent state for the target temperature has been reached and the evolutionary procedure should be terminated. To continue the iterations beyond such a convergent state will yield little or no improvement in the design objective.

The evolutionary iteration procedure for minimizing the nodal temperature is re-organized systematically as follows:

Step 1:
Discretize the heat conduction field using a dense FE mesh ($\mathbf{E}$), in which the element sets in the design and non-design domains are denoted by $\mathbf{D}$ and $\mathbf{N}$ respectively, $\mathbf{D} \cap \mathbf{N} = \emptyset$ and $\mathbf{D} \cup \mathbf{N} = \mathbf{E}$. Assign the initial design variable of elements in the design domain to $\eta = 1$, and define ESO driving parameters $\mu^{(k=0)}$ and $\Delta\mu$.

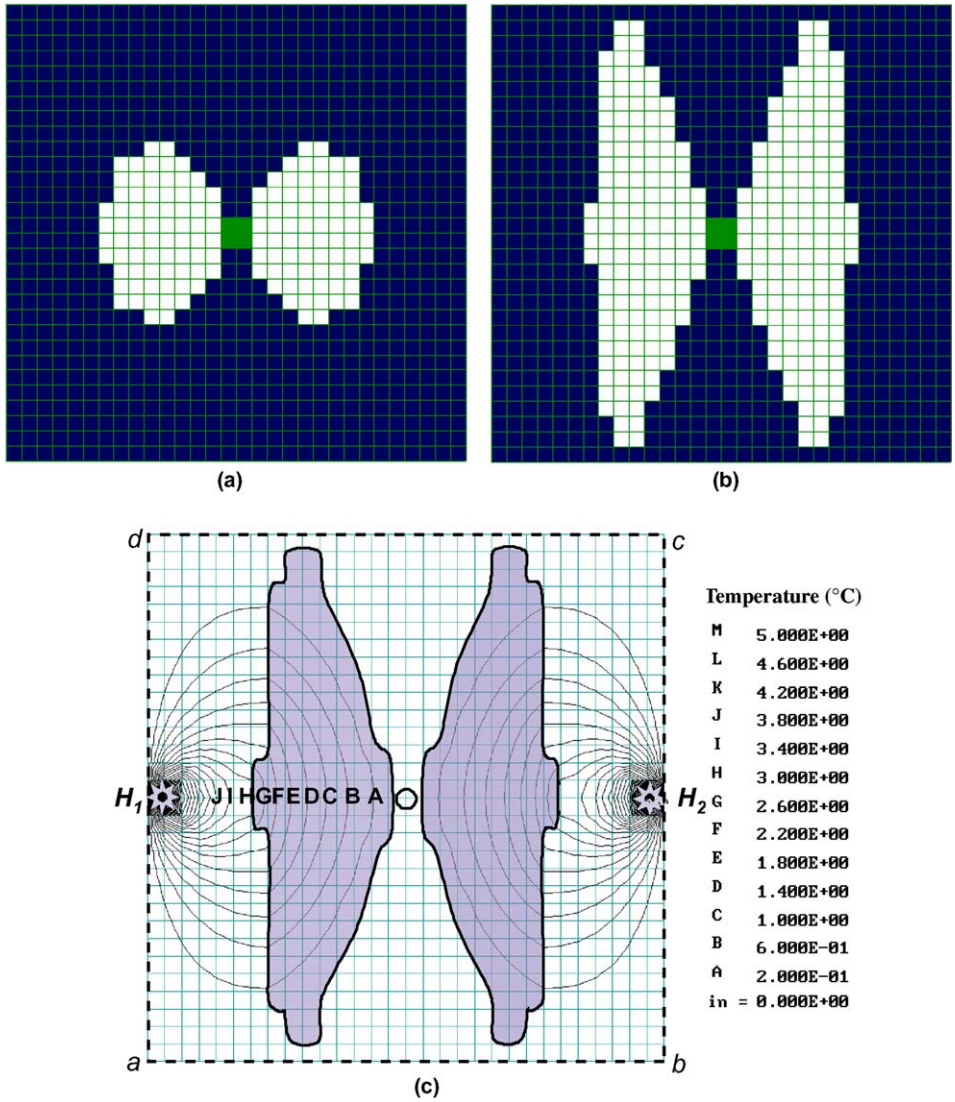

Fig. 6. Initial design domain and temperature distribution with two heat sources: (a) intermediate topology of insulators ($k = 3$), (b) optimal topology of insulators ($k = 7$) and (c) temperature contour with the optimal design of insulators.

Step 2:
Perform a FEA for the real (2) and fictitious thermal systems (9) respectively.
Step 3:
Compute temperature sensitivity α_T^e for all $e \in \mathbf{E}$ by using Eq. (11).
Step 4:
Remove or degenerate a number of elements $e \in \mathbf{D}^{-(k)}$ which satisfy both Eqs. (16) and (17) and then set $e \in \mathbf{N}$.
Step 5:
If a steady state is reached, increase $\mu^{(k)}$ by $\Delta\mu$, as Eq. (18), and set $k = k + 1$, repeats Step 4; Otherwise, repeat Steps 2–4 until Eqs. (19) or (20) is satisfied.

As pointed out by Garreau et al., Céa et al. and Guillaume and Idris [28,39,40], in essence, the evolutionary procedure can be seen as a steepest descent method where the descent direction is determined by the temperature sensitivity α_T^e and the step length is controlled in terms of the *evolutionary rate* $\Delta\mu$. To maintain a smooth change between two consecutive iterations, in general, the evolutionary rate is prescribed at a small percentage. A more detail can be found from the monograph by Xie and Steven [23]. As an alternative, the step length can also be controlled by either a fixed *removal ratio* $\Delta\upsilon_f$ of volume (the ratio of current removed volume to the initial volume) [32,38] or a varying *removal ratio* $\Delta\upsilon_v$ of volume (the ratio of removed volume at the current step to volume of the previous step) [28,31,39,40]. In general, the fixed removal ratio is set at a lower percentage (say $\Delta\upsilon_f = 1$–3% as of [23]) and the varying removal ratio can be set to a higher percentage (say $\Delta\upsilon_v = 5$–10% as of [28,40]). In numerical computation, the value of the *evolutionary rate* $\Delta\mu$ or *removal ratio* $\Delta\upsilon$ can be appropriately adjusted by monitoring the change in the objective function or geometrical connectivity being generated.

4. Illustrative examples

The following examples are used to demonstrate the capabilities of the proposed evolutionary topology optimization method for solving temperature control problems. It is assumed that all examples are subject to two-dimensional steady heat conduction, where thermal convection and radiation are not considered (even for some newly created free boundaries). For the simplicity of finite element modeling, the examples are meshed using 2D four node quadrilateral elements with a unit thickness.

The first example has those negatively sensitive elements progressively removed from the design domain. As a consequence of this, an optimal topology is generated in the conductive field, where the target temperature at the controlled point is minimized. The second example aims at determining the optimum topological distributions of insulation materials. In the design processes, the initial conductive material is gradually converted to the insulators so as to reduce the objective temperature at the specified position(s).

4.1. Topological design of conductive field

A heat conducting field with the dimension of 30 mm × 30 mm is meshed in 30 × 30 four node square elements. The boundary temperature along the top (*dc*) and the left edges (*da*) is maintained at $T_\Gamma = 0$ °C (273 °K), while the temperature at the lower right corner (point *b*) is set at $T_b = 1$ °C (274 °K), as illustrated in Fig. 2(a). There is no heat flux assumed along all these four boundaries (i.e. edges *ab*, *bc*, *cd* and *da*).

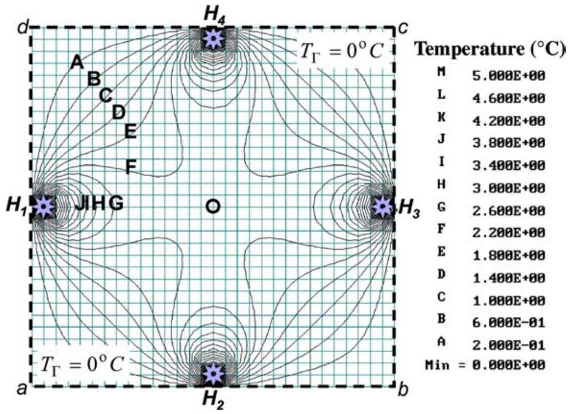

Fig. 7. Initial design domain and temperature contour with four heat sources.

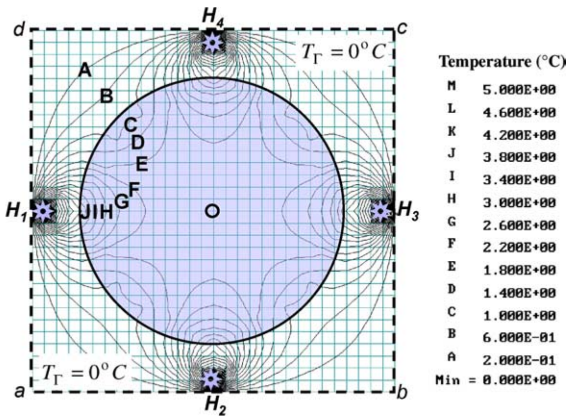

Fig. 8. Traditional design of insulation (with a target temperature at $J_O = 1.128$ °C).

In this example, the minimization of the temperature at central point O of the square field is sought. Those four elements that directly connect to this node (O) are considered as the non-design domain **N** (as shaded in Fig. 2(a)). In the design process, an evolutionary rate of $\Delta\mu = 2\%$ is set. As more and more elements with the most negative sensitivities are removed, the temperature J_O at the controlled point O is progressively approached to zero as plotted in Fig. 3.

Corresponding to the final stage of temperature minimization (at steady state 7), Fig. 2(b) shows the optimized topological design. It is interesting to note that a topological pattern with 45° corner connection of the elements can be observed in the farthest end to the lower right corner node that has the highest temperature in the field. This simply forms a *longest* conductive path from the high temperature point to the controlled point.

4.2. Designs of thermal insulation topologies

The second example is employed to show some topological design problems of heat insulation materials in conductive fields. A region of 30 mm × 30 mm is taken into account here, in which the temperature in those

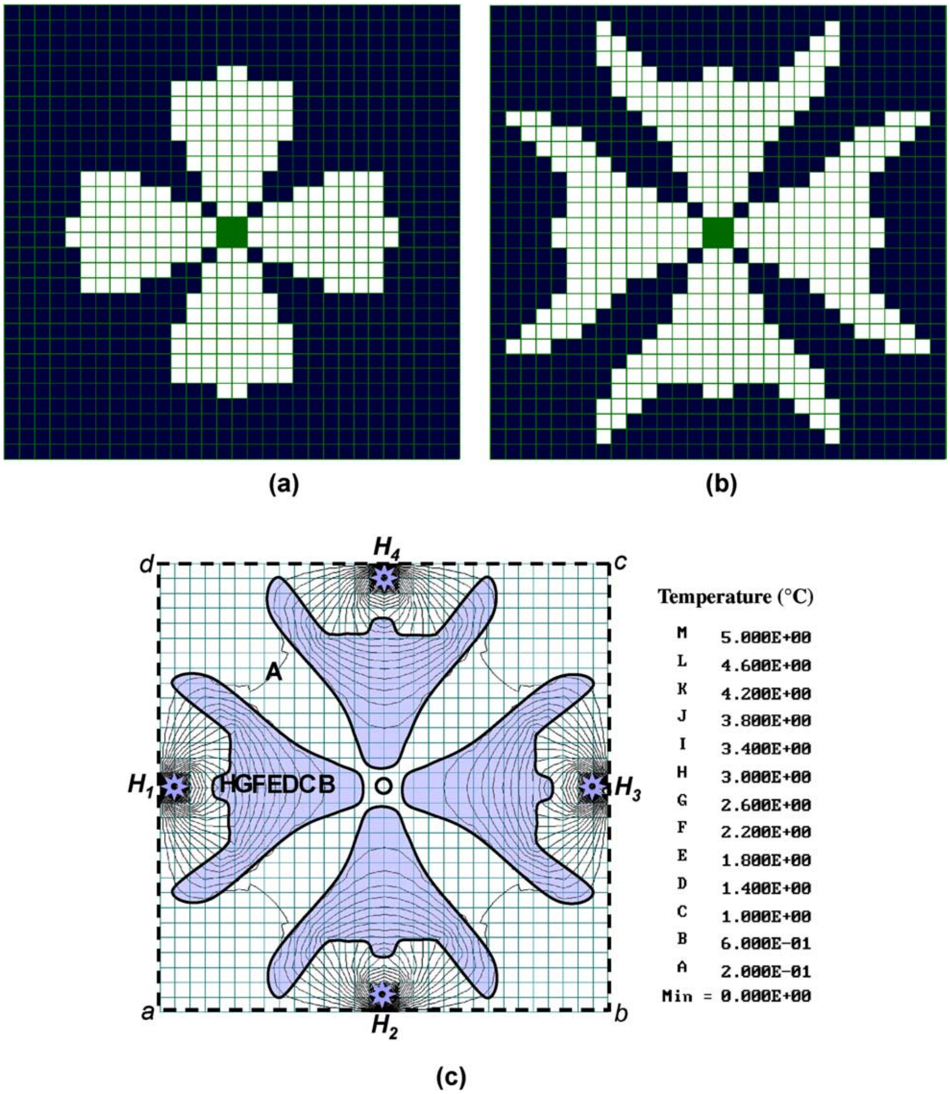

Fig. 9. Initial design domain and temperature distribution with two heat sources: (a) intermediate topology of insulators ($k = 2$), (b) optimal topology of insulators ($k = 8$) and (c) temperature contour with the optimal design of insulators (with the objective temperature of $J_O = 0.292$ °C).

four outer boundaries is kept at 0 °C (273 °K). The conducting fields to be designed consist of two materials, one for conductor with a thermal conductivity of $\kappa_2 = 0.045$ W/mm K and another for insulation with a conductivity of $\kappa_1 = 0.001$ W/mm K (i.e. $\kappa_1 \approx 2\%\kappa_2$). To observe the variation of topologies of different design cases, the two and four heat sources (with a higher nodal temperature at each heat source H) are considered respectively in the example.

Fig. 4 depicts the temperature distribution for the initial design, where two heat sources (H_1 and H_2) with a temperature at $T_{H1} = T_{H2} = 10$ °C (283 °K) are set. Initially, the design domain **D** is made of pure conductive material (with the conductivity of $\kappa_2 = 0.045$ W/mm K). It can be seen that, from the temperature contour illustrated in Fig. 4, the objective temperature then has a level at $J_O = 1.22$ °C (274.22 °K). In the evolution-

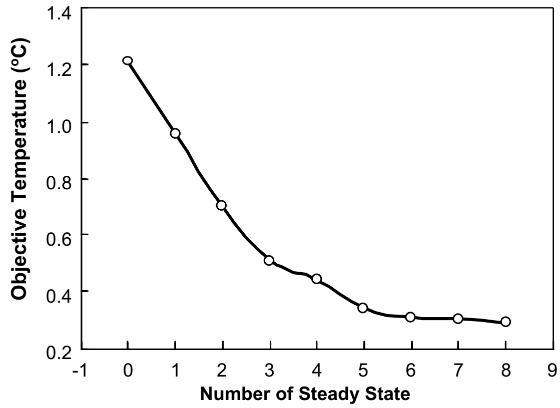

Fig. 10. The evolutionary history of objective temperature with four heat sources.

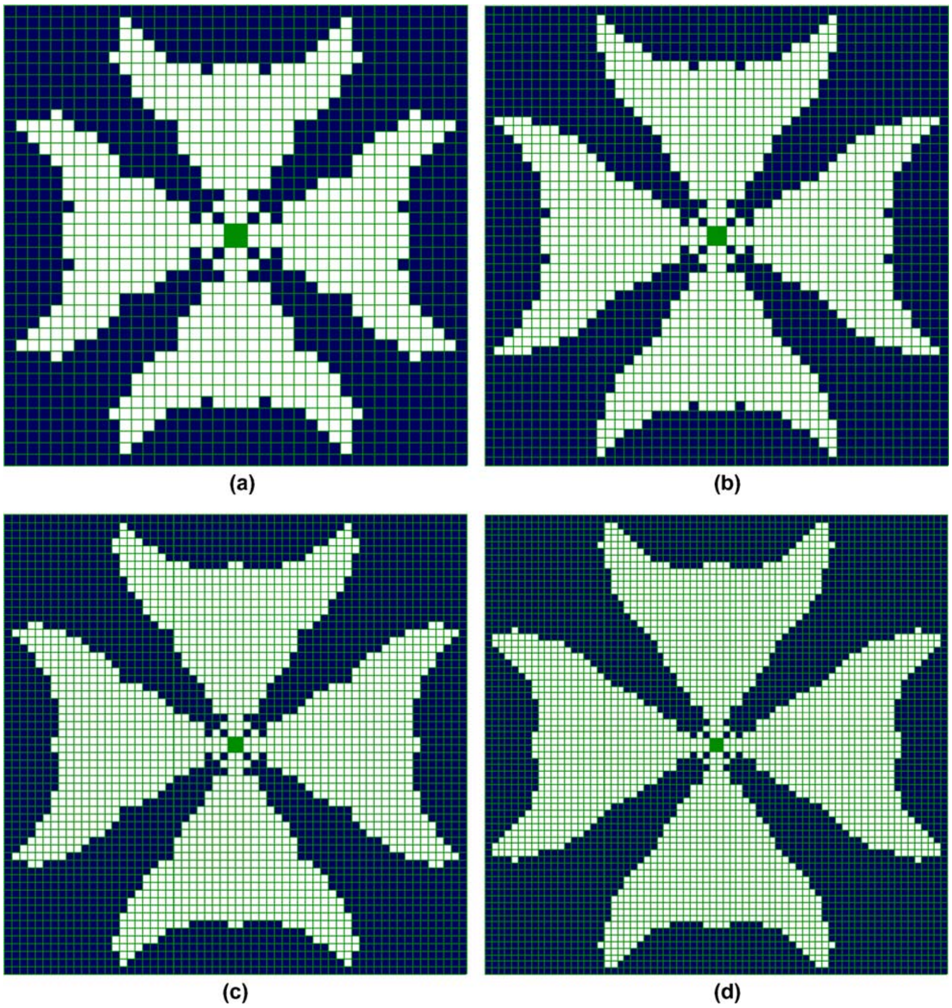

Fig. 11. The mesh effect on the topological designs of insulators: (a) Mesh 40 × 40, (b) Mesh 50 × 50, (c) Mesh 60 × 60 and (d) Mesh 70 × 70.

ary optimization process, as some negatively sensitive conduction material is progressively converted to insulator, the objective function is gradually reduced towards a much lower level at $J_O = 0.0185$ °C (273.0185 °K), as plotted in Fig. 5. This represents a 98.5% reduction in the objective temperature.

An intermediate and the final topologies of the insulation material are shown in Figs. 6(a) and (b), and the optimized temperature contour is depicted in Fig. 6(c). It is clearly seen that the optimal topology of insulator material efficiently isolates the controlled point O from the heat conduction. Obviously, the topology design of the insulator forms the best possible barrier to prevent heat from high temperature zones to the targeted temperature control point.

Fig. 7 shows the initial temperature contour with four heat sources at a higher temperature of $T_{H1} = T_{H2} = T_{H3} = T_{H4} = 10$ °C (283 °K) and the zero-temperature at the boundaries (*ab*, *bc*, *cd* and *da*). When there are no insulators distributed initially, the objective temperature at the center of the analyzed region is presented as $J_O = 1.21$ °C (274.21 °K), as in Fig. 7. For such a surrounding multi-heating case, one can intuitively think of a more conventional design with the circular shaped insulator, as illustrated in Fig. 8, in which the controlled point is completely isolated by insulation material from the surrounding high temperature zones. To make a comparison possible, a volume constraint of $V_0 = V_1 = 340$ mm^3 (insulator) or $V_2 = 900$ mm^3 (conductor), i.e. $V_1/V_2 = 60\%$, is prescribed to allocate the circularly shaped insulator and conducting materials. As a consequence of such a circular-shielded design, the objective function can be reduced to a moderate level of $J_O = 1.128$ °C (274.128 °K) as in Fig. 8.

When the evolutionary procedure is applied, the insulation material is optimally distributed over the design domain **D** as more and more negatively sensitive conducting elements are converted to insulation ones, as presented in Fig. 9(a) and (b). It is interesting to note that, although there still exist some conductive channels from the high temperature zones to the controlled point, the target temperature has been decreased to $J_O = 0.292$ °C (273.292 °K), a 76% reduction from the non-insulation design and a 74% reduction from the conventionally circular insulator design. Shown in Fig. 10 is the evolutionary history plot of the objective temperature.

The temperature contour of the optimum topology is depicted in Fig. 9(c). Compared this with those in the non-insulation (Fig. 7) and the circular designs (Fig. 8), it can be found that the optimal topology provides a much more uniform temperature descents (or negative gradients) inside the insulators. This indicates that the insulation material in optimum design plays a more efficient role on heat shielding.

To clarify the effect of finite element mesh on the resulting topologies, Figs. 11(a)–(d) gives a comparison in several different mesh densities of 40×40, 50×50, 60×60 and 70×70. From this example, it seems that the mesh densities do not lead to noticeable changes in the topologies, nor does there exist significant checkerboard pattern compared to elasto-static problems.

5. Concluding remarks

This paper focuses on the topology design methods and applications for thermally conductive materials. Unlike the shape-driven design methodology developed in the existing literature, the present evolutionary procedure does not require any initial hole or seed insulator for seeking a new material topology. In this study, the design variable is constructed in terms of an element's thermal conductivity.

As distinguished from previous studies in non-gradient based topology algorithm presented by the authors, this piece of work derives a discrete sensitivity formulation with respect to element's absence or material degeneration in a heat finite element framework. According to the negativity or positivity of the temperature sensitivity, a conducting field is divided into two complementary regions. Removing or degenerating element's materials from the most negative sensitive region will result in a most efficient temperature reduction at a specific targeting node. More importantly, the temperature sensitivity provides a great potential to some more extensive design objectives in thermal engineering. From the experience gathered in [25,26], it can be claimed that the temperature sensitivity-driven evolutionary procedure offers a new capability to the topological design of other general field problems governed by the quasi-harmonic equation.

To seek an optimum, the ESO method is employed in this paper. The ESO method is implemented from a more intuitive concept that by progressively removing or degenerating conductive material at the deepest decent direction, the residual topology evolves towards a status with a higher thermal performance. It can been seen that from this article the extension of the well-established ESO method to heat conducting problem is quite straightforward and does not increase any computational complexity.

It is worth pointing out that however, the temperature sensitivity does not take into account the effects of perturbation of design variables on the change of the applied thermal loading, i.e. $\Delta \boldsymbol{Q}$ is neglected in Eq. (6). This means that the topology described by voids or holes ($\kappa_1 = 0$) only reflects an optimal design for the pure *conducting* performance. In practice, one should take care of the formation of new boundaries, where the convection and even radiation need being carefully considered. Obviously, a more specific study is needed to address this issue in future [30].

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